2.1
Review adding and subtracting integers:

\[-7 + (-3) \quad 9 + (-2) \quad -8 + 5 \quad 17 + (-11) + (-15)\]

\[6 - (-3) \quad 6 + (-10) \quad -10 - 5 \quad 20 + (-1) - (-4)\]

**Solving Equations with the Additive Property of Equality**

Addition Property of Addition: If \( a = b \), then \( a + c = b + c \).

- Your goal is the statement \( x = \# \)
- Isolate the variable by applying the opposite operation of the constant. This is using the additive inverse.
- Check your solution by substituting your answer back into the original equation and confirming that it is correct.

**Examples:**

\[x - 2 = 1\]

\[x + 1.5 = -4.5\]

\[x + 4 = 0\]

\[-31 = -10 + x\]

\[x + \frac{2}{5} = \frac{7}{10}\]

\[\frac{1}{3} + x = \frac{3}{4}\]
2.2 day 1 Solve Equations with the Multiplicative Property of Equality

Review Multiplication and Division of Rational Numbers

\[
\begin{array}{cccc}
-4(-5) & 7(1) & -9(7) & 5(-19) \\
10 & -7 & 100 & -3 \\
-2 & 14 & 20 & -1 \\
\end{array}
\]

Multiplicative Property
If \( a = b \), then \( ac = bc \)

Solving equations using multiplication or division uses the same rules as solving with addition or subtraction. To remove multiplication, you use the opposite operation... division. Your goal is still the statement \( x = \# \).

\[
\begin{align*}
3x &= 27 & -7x &= 14 & -15 &= -3x \\
19 &= -5x & -0.04x &= -4.8 \\
\end{align*}
\]

Concept Check

\[
\begin{align*}
1.2 &= 6x & -2x &= 10 & -0.13x &= -0.78 \\
\frac{x}{2} &= 3 & \frac{x}{5} &= -7 & -\frac{x}{6} &= -\frac{1}{3} \\
\end{align*}
\]

Concept Check

\[
\begin{align*}
-\frac{1}{4}x &= \frac{3}{8} & -\frac{2}{5}y &= -\frac{15}{4} \\
\end{align*}
\]
2.2 day 2 Solving Multi-Step Equations Part I

The following problems will utilize both the addition and multiplication properties of equality. The goal is to solve for $x$ and get $x = \text{a number}$.

- Remove any grouping symbols such as parentheses.
- Simplify each side by combining like terms.
- Get all variable terms on one side and all numbers on the other side by using the addition property of equality.
- Get the variable alone by using the multiplication property of equality.
- Check the solution by substituting it into the original equation.

\[ 7(x - 5) = 3 \quad 2x + 5 = 11 \]

\[ -4x + 3 = -33 \quad 7 = 3 - \frac{5x}{4} \]

\[ -6x - 9 = -21 \quad \frac{2}{7}y - \frac{1}{5} = 2 \]

Concept Check

\[ 12 = 4 + 4x \quad -9.0 + 0.2x = -5.5 \]
2.2 day 2 Solving Multi-Step Equations Part II

$4x + 5 - 7x + 3 = 10x - 2 - 11x - 6$  
$4(11 - 6x) = 10 - 7x$

$-8.1x + 3(x - 2) = 9(2.4 + 0.05x)$  
$5(x - 3x) - 2 = 5x - 2(2x - 10) + 10$

Concept Check

$6 + 2(3 - 8x) = -14 + 2x$  
$-4(x + 3) - 4 = -7(8)$

Write the following as algebraic expressions

A 20 foot board is cut into 2 pieces. If one piece is $x$ feet long, write an expression for the length of the other piece.

The total cost of 2 books is $58. If one book costs $x$ dollars, write an expression for the cost of the other book.
2.2 day 2 Solving Multi-Step Equations Part II

Consecutive numbers
What are consecutive numbers?
   Examples:

If the first number is $x$, what is the next consecutive integer?

What are even consecutive numbers?
   Examples:

If the first number is $x$, what are the next two even consecutive integers?

If $x$ is the first of two consecutive integers, what is the sum of the first two numbers?
   first:
   second:
   sum of the two consecutive integers:
2.4 Applications of Linear Equations

Number problems:
1. Identify the unknown and call it x
2. Write an equation using x and other relevant numbers in the problem
3. Solve for x
4. Check to make sure the answer makes sense

Two numbers add to 20 and the first number is 4 more than the second number. Find the 2 numbers.
   a. Label the variable.
   b. Write an equation to solve the problem.
   c. Solve the equation.
   d. State the answer in a sentence.

Your cell phone plan charges $50 per month plus 0.20 per text. How many texts can you send/receive if your budget is $85?
   a. Label the variable.
   b. Write an equation to solve the problem.
   c. Solve the equation.
   d. State the answer in a sentence.
2.4 Applications of Linear Equations
One number is 8 times another number. If the sum of the numbers is 63, what are the numbers?

a. Label the variable.

b. Write an equation to solve the problem.

c. Solve the equation.

d. State the answer in a sentence.

Consecutive Integer problems:
When asked to find consecutive integers, call the first one $x$, then use $x$ to describe the others:

With a partner complete the table below

<table>
<thead>
<tr>
<th>Phrase</th>
<th>Label for 1st Integer</th>
<th>Label for 2nd Integer</th>
<th>Label for 3rd Integer</th>
<th>Indicated Sum (simplify the expression)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of the first three consecutive integers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum of three consecutive even integers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum of two consecutive odd integers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum of the second and third consecutive integers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Find three consecutive integers that add up to 18

a. Label the variable.

b. Write an equation to solve the problem.

c. Solve the equation.

d. State the answer in a sentence.
2.5 Formulas and Solving for a Variable

Solving a formula for a specific variable requires the formula to be rearranged. Your answer will still contain all the original variables. Solving for a specific variable follows the same steps as solving for $x$ in 2.1 and 2.2 did.

Solve $C = 2\pi r$ for $r$ \hspace{2cm} Solve $2x + y = 6$ for $y$

Solve $A = \frac{1}{2}bh$ for $h$ \hspace{2cm} Solve $V = \frac{2}{3}\pi r^2 h$

Concept Check
Solve $P = a + b + c$ for $b$ \hspace{2cm} Solve $S = 4lw + 2wh$ for $h$

Converting between Celsius and Fahrenheit $F = \frac{9}{5}C + 32$
Convert 90 degrees Fahrenheit to Celsius.

Concept Check
Convert 23 degrees Fahrenheit to Celsius.

Story problems:
A volume of a cone is 565.2 cubic centimeters. If the radius of the cone is 6 centimeters, what is the height? Round to the answer to the hundredths place.

a. Label the variable.

b. Write an equation to solve the problem.

c. Solve the equation.

d. State the answer in a sentence
2.5 Formulas and Solving for a Variable

A Japanese “bullet” train set a new world record for train speed at 552 kilometers per hour on the Yamanashi Maglev Test Line, which was 42.8 kilometers long. How many minutes did it take for the train to complete this record? Round the answer to the nearest hundredth of a minute.

a. Label the variable.

b. Write an equation to solve the problem.

c. Solve the equation.

d. State the answer in a sentence

Concept Check

The area of a dog kennel is 45 square feet. If the length of the kennel is 9 feet, what is the width?

a. Label the variable.

b. Write an equation to solve the problem.

c. Solve the equation.

d. State the answer in a sentence.
2.7 Linear Inequalities

<table>
<thead>
<tr>
<th>Expression</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &lt; 3$</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>$x \leq 3$</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>$-4 &lt; x &lt; 3$</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>$-4 &lt; x \leq 3$</td>
<td><img src="image" alt="Graph" /></td>
</tr>
</tbody>
</table>

**Solving inequalities**
Solve as if it is an equation with one exception: *if you multiply or divide both sides by a negative number, switch the inequality so it points the other way.*

We will also ask you to write your solution in set notation. An example looks like this: \{x \mid x > 3\} and reads "The set of all x such that x is greater than 3".

Example:

1. $-3x + 2 < -x + 8$
   
   ![Graph](<image>)
   
   i. Set notation:

2. $\frac{2}{3}x > 3$
   
   ![Graph](<image>)
   
   i. Set notation:
3. \(-5x < 15\)

   i. Set notation:

4. \(-3(2x - 5) < 3(x - 1)\)

   i. Set notation:

Concept Check

5. \(x - 5 > -7\)

   i. Set notation:

6. \(-x < 0\)

   i. Set notation:
3.1 Ordered Pairs and Quadrants

\[ x - \text{axis: the horizontal axis, independent variable} \]
\[ y - \text{axis: the vertical axis, dependent variable} \]

Ordered pair: the way a point is described
\[(x, y)\]

Example: Plot the points and label.

A: \((1, 4)\)
B: \((4, -6)\)
C: \((-4, -3)\)
D: \((0, 7)\)
E: \((-5, 0)\)
F: \((-2, 1)\)

Concept Check:
What is the ordered pair for the point where the \(x\)- and \(y\)-axis intersect?

The name for this point is *the origin.*

Quadrants:
The four areas of the Cartesian Coordinate System are referred to as quadrants. They are labeled with Roman Numerals.

II
\[ \begin{array}{c|c}
\hline
II & I \\
\hline
III & IV \\
\hline
\end{array} \]

Label which quadrant each point above is in.

A _______  D _______
B _______  E _______
C _______  F _______
3.4 Ordered Pairs and Slope

Ordered Pairs and Slope

- Plot $(6, 2)$ and $(-1, 4)$.

  What is the vertical difference between the points?

  What is the horizontal difference between the points?

  Draw a line through the points.

  What is the slope of the graph?
  \[ m = \frac{\text{rise}}{\text{run}} = \]

- Plot $(-3, 6)$ and $(-1, -2)$.

  What is the vertical difference between the points?

  What is the horizontal difference between the points?

  Draw a line through the points.

  What is the slope of the graph?
  \[ m = \frac{\text{rise}}{\text{run}} = \]

Slope Formula with Two Points

\[ m = \frac{Y_2 - Y_1}{X_2 - X_1} \text{ or } \frac{\text{difference of y-values}}{\text{difference of x-values}} = \frac{\Delta y}{\Delta x} \]

Examples: Calculate the slope.

A) $(4, 5)$ and $(-1, 3)$

B) $(9, -1)$ and $(-3, 5)$
Slope – intercept form

\[ y = mx + b \]

- \( b \) is the starting point on the \( y \)-axis for graphing. It is officially known as the \( y \)-intercept.

- \( m \) is the slope of the graph. It provides direction and a numerical description of how steep a line is.
  - If \( m \) is positive, the graph is rising to the right. \( \uparrow \)
  - If \( m \) is negative, the graph is falling to the right. \( \downarrow \)

Example: Describe the graphs as having a positive or negative slope.

a) \[ \] b) \[ \] c) \[ \]

- \( m \) is ______
- \( m \) is ______
- \( m \) is ______

Example: Identify the starting point and the slope

a) \( y = 5x + 1 \) \hspace{1cm} b = ______  \hspace{1cm} m = ______

b) \( y = -7x + 4 \) \hspace{1cm} b = ______  \hspace{1cm} m = ______

c) \( y = \frac{2}{3}x - 3 \) \hspace{1cm} b = ______  \hspace{1cm} m = ______

d) \( y = -\frac{1}{4}x - 5 \) \hspace{1cm} b = ______  \hspace{1cm} m = ______

e) \( y = x + 4 \) \hspace{1cm} b = ______  \hspace{1cm} m = ______

f) \( y = -x \) \hspace{1cm} b = ______  \hspace{1cm} m = ______

Concept Check: Could the graph of \( y = 2x + 1 \) be ?

Why?
Slope has two parts; the rise and the run. The graph below has a slope equal to 1/3.

Rise: vertical movement (up or down)

Run: horizontal movement (left or right)

Graph the equations

a) \( y = 5x + 1 \)
   \[
   b = \_
   \]
   \[
   m = \_
   \]
   The rise is \_
   The run is \_
   \( m \) is \_
   the graph is \( \uparrow \text{ or } \downarrow \)

b) \( y = \frac{2}{3}x - 3 \)
   \[
   b = \_
   \]
   \[
   m = \_
   \]
   The rise is \_
   The run is \_
   \( m \) is \_
   the graph is \( \uparrow \text{ or } \downarrow \)
3.5a Graphing with Slope Intercept Form

C) \( y = -x \)
   \( b = \) _____
   \( m = \) _____
   The rise is _____.
   The run is _____.
   \( m \) is __________; the graph is \( \nearrow \) or \( \searrow \)

D) \( y = -\frac{1}{4}x - 5 \)
   \( b = \) _____
   \( m = \) _____
   The rise is _____.
   The run is _____.
   \( m \) is __________; the graph is \( \nearrow \) or \( \searrow \)

Concept Check: Match the graphs with their equations.

\[ y = -\frac{1}{2}x + 2 \]
\[ y = -2x + 2 \]
\[ y = 2x + 2 \]
Rearranging Equations Into Slope-Intercept Form

- Move the x-term with addition or subtraction.
- Divide all the terms by y's coefficient.

Examples: Solve the following for y

a) \( x + 2y = 6 \)  
   b) \(-3x + 7y = 0\)  
   c) \(-y - 5x = 14\)

Example: Graph \( y + 5 = -\frac{1}{2}x \)

Concept Check:

a) What is the y-intercept and slope of \( 8x + 4y = 12 \)?

b) Graph \( 8x + 4y = 12 \)
Graphing with Standard Form \( AX + BY = C \)

The equation \( 3X - 4Y = 12 \) is graphed to the right.

What is the \( x \)-intercept? 
( , )

What is the \( y \)-intercept? 
( , )

Look back at the intercepts and the equation. How can the intercepts be calculated with division?

Example: Graph \( X + 2Y = 6 \) with the intercepts.

What is the \( x \)-intercept? 
( , )

What is the \( y \)-intercept? 
( , )

Concept Check: Use the equation \(-8X - 2Y = 16\) to complete each part below.

a) What are the \( x \)- and \( y \)-intercepts of the equation?

b) Determine the slope of the equation with two different methods.

i) Calculate the slope with the formula \( \frac{y_2 - y_1}{x_2 - x_1} \) by using the \( x \)- and \( y \)-intercepts in part (a).

ii) Rearrange the equation into slope-intercept form to verify your slope is correct in part (i).
3.3 b & c Vertical and Horizontal Graphs

Vertical Graphs and Their Equations

Label at least three ordered pairs of the graph to the right.
Notice that all the points have the same x-value.
What x-value do all the points have? _______

The equation for this graph is ____________________.

The slope of all vertical graphs is ________________.
Verify this by selecting two of the ordered pairs and calculating the slope.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

Example: Graph \( x = -2 \) and state the slope.

Concept Check

a) Graph \( x = -4.5 \)
b) What is the x-intercept?
c) What is the y-intercept?
d) What is the slope?
e) Can \( x = -4.5 \) be written in slope-intercept form?
3.3 b & c Vertical and Horizontal Graphs

Horizontal Graphs and Their Equations

Label at least three ordered pairs of the graph to the right.
Notice that all the points have the same y-value.
What y-value do all the points have? ______

The equation for this graph is ________________.

The slope of all vertical graphs is ________________.
Verify this by selecting two of the ordered pairs and calculating the slope.

\[ m = \frac{y_2-y_1}{x_2-x_1} \]

Example: Graph \( y = 1 \) and state the slope.

Review

a) Graph \( 3x + 5y = -5 \) by changing it to slope-intercept form.
b) What is the y-intercept?
c) What is the slope?
d) What is the x-intercept?
e) The line you graphed passes through many points (ordered pairs). The x- and y-intercepts are points that the graph passes through. Write another ordered pair that the line passes through.
Review Slope, Slope-Intercept Form, and Standard Form

a) Graph \( y = -x - 3 \)

b) Graph a line passing through \((-3,1)\) and \((2,-4)\).

c) Graph \(-2x + 4y = -8\) by finding the \(x\) and \(y\)-intercepts.

d) Graph \(x = 4\)

e) Use the graph and ordered pairs in problem (b) to write the equation for the graph.

f) Convert \(-2x + 4y = -8\) to slope-intercept form. Then make sure the graph in problem (c) has the slope and \(y\)-intercept that it should have.
Parallel and Perpendicular

Parallel lines do not intersect. They have the same slope. If one equation is \( y = 2x + 3 \) then all graphs parallel to this also have a slope of 2.

Example
a) Graph \( y = -\frac{1}{3}x - 1 \)

b) Write the equations of two graphs parallel to the graph of (a).
   i. 
   ii. 

c) Graph the equations in part (b).

d) Write the equation of a graph parallel to \( y = -\frac{1}{3}x - 1 \) with a \( y \)-intercept of (0,9).

Perpendicular lines intersect at a \( 90^\circ \) angle. Their slopes are negative reciprocals of each other. If a slope is \( \frac{5}{3} \) then the slope of a perpendicular graph is \( -\frac{3}{5} \).

Example: Write the slopes of lines that would be perpendicular to the given slopes.
   i) \( \frac{3}{2} \)   ii) 4   iii) \( \frac{-2}{5} \)   iv) \( \frac{-1}{7} \)

Examples
a) What is the slope of a graph perpendicular to \( -\frac{1}{4}x + y = 0 \)?

b) What is the slope of a graph perpendicular to \( 3y + 4x = 6 \)?

c) Graph \( y = 3x - 4 \) and \( 2x + 6y = 12 \).
   
i. From the picture are the two graphs perpendicular?
   
ii. How else can you determine that the lines are or are not perpendicular?
3.4 & 3.5b Parallel and Perpendicular Lines

More Parallel, Perpendicular, Horizontal, and Vertical Graphs

Recall: Vertical graphs have an undefined slope and horizontal graphs have a slope of zero.

Example

a) Graph \( x = 2 \). Then graph a parallel line through \((0, -1)\). Write the equation of the parallel line.

b) Graph \( y = -4 \). Then graph a perpendicular line through \((5,3)\). Write the equation.

c) Write an equation with a slope of zero through \((-2.5, 7)\).

d) Write an equation through \((3, -1)\) parallel to \( y = -0.5 \).

Review

i. Plot the points \((2,2)\) and \((7,3)\).

ii. Draw a line through the points.

iii. Calculate the slope with \( m = \frac{y_2 - y_1}{x_2 - x_1} \).

iv. Write the equation of the line you drew.

v. Write the equation of the line parallel to (iv) and has a \( y \)-intercept of 5.
3.6 Graph Inequalities

Graphing inequalities is very similar to graphing linear equations, except half of the graph is shaded to indicate the less than or greater than. Follow these steps to graph inequalities:

To graph inequalities:
1. graph as if it is an equation.
2. draw a solid line for ≤ and ≥
3. draw a dotted line for < and >
4. pick a point on either side of the line and substitute in x and y in the inequality
5. if true, shade where that point is; if false shade the other side (where the point is not)

Example:
y > 2x

\[
\begin{array}{c|c|c}
\text{x} & \text{y} & \text{y} > 2x \\
0 & 0 & \\
1 & 2 & \text{true} \\
-1 & -2 & \text{false}
\end{array}
\]

I picked (0,2) as a test point.
When I plugged this into the original inequality I got 2 > 2(0) which is the same as 2 > 0.
Since this is a true statement I shaded the side of the line that contained (0,2).

Graph the following:

\[x + 3y \geq 7\]
3.6 Graph Inequalities

\[ y \geq 4 - 2x \]

\[ y \leq 5 \]

\[ x > 3 \]
A system of linear equations consists of two or more linear equations. In this section, we focus on solving systems of linear equations containing two equations with two variables.

### Types of Answers to a System of Equations

**One point of intersection**  
One Solution

**Parallel** - No Solution

**Same Line** - Infinite Solutions

<table>
<thead>
<tr>
<th>Type:</th>
<th>Consistent</th>
<th>Inconsistent</th>
<th>Dependent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution:</td>
<td>$(x, y)$</td>
<td>No solution</td>
<td>Infinite solutions ${(x, y)</td>
</tr>
</tbody>
</table>

### Deciding Whether an Ordered Pair Is a Solution

To decide if an ordered pair is a solution to a system of equations, substitute the $x$-value and the $y$-value of the ordered pair into both equations. If both equations make a true statement, then the ordered pair IS a SOLUTION to the system of equations.

1. Determine the solution of the system of equations represented by the graphed equations.
   
   \[
   \begin{align*}
   y &= 3x - 9 \\
   2x + y &= 1
   \end{align*}
   \]

2. An ordered pair is a solution to a system if the ordered pair is true in all the equations. 
   Determine if $(1, 6)$ is a solution to the system.
   
   \[
   \begin{align*}
   5x - y &= -1 \\
   x + 5 &= -6
   \end{align*}
   \]
Solving Systems of Equations by Graphing

Solve the following systems of equations by graphing and determine the type of system.

3. \begin{align*}
2y &= -4x \\
2x + y &= 7
\end{align*}

*The equations in #3 have the same slope and different y-intercepts. The system is _______________ and there is _____ solution.

4. \begin{align*}
x + y &= -4 \\
-2y &= 2x + 8
\end{align*}

*The equations in #4 have the same slopes and the same y-intercepts. The system is _______________ and there are ____________________.

5. \begin{align*}
y &= 2x + 5 \\
x &= -3
\end{align*}

*The equations in #5 have different slopes. The system is _______________ and the solution is an _______________.

4.1: Solve Systems of Linear Equations by Graphing

Concept Check
6. Determine the solution and type of system based on the picture to the right.

7. Solve the system of linear equations by graphing, state the ordered pair and the type of system.
\[y = -x + 2\]
\[-x + y = 4\]

Instructor: Application
8. A circus show charges $4.25 per child and $8.50 per adult. If $68 was spent by a group of 10 on circus tickets, how many adult tickets were purchased?

Variables: 
\[x = \text{number of children} \]
\[y = \text{number of adults} \]

Equation 1: 

Equation 2: 

There were __________ adult tickets purchased.
4.2: Solve Systems of Linear Equations by Substitution

Steps for Solving a System of Linear Equations with the Substitution Method.
Step 1: Solve one of the equations for one of its variables.
Step 2: Substitute the expression for the variable found in Step 1 into the second equation.
Step 3: Solve the equation from Step 2 to find the value of one of the variables.
Step 4: Substitute the value found in Step 3 into any equation containing both variables to find the value of the second variable.
Step 5: Check the proposed ordered pair solution in the original system.

Example Problem Worked Out
\[
\begin{align*}
2x + y &= 10 \\
x - y &= 2
\end{align*}
\]

Step 1: Take \( x - y = 2 \) and solve for \( x \).
\[
x = y + 2
\]

Step 2: Substitute \( y + 2 \) into \( 2x + y = 10 \) where \( x \) is.
\[
2(y + 2) + y = 10
\]

Step 3: Solve \( 2(y + 2) + y = 10 \) for \( y \).
\[
2y + 4 + y = 10 \\
3y + 4 = 10 \\
3y = 6 \\
y = 2
\]

Step 4: Substitute \( y = 2 \) into \( x - y = 2 \) to determine \( x \).
\[
x - 2 = 2 \\
x = 4
\]
Solution is \((4, 2)\)

Step 5: Check the solution in the original system.
Equation 1: \( 2(4) + 2 = 10 \)
Equation 2: \( 4 - 2 = 2 \)

Solve the system of equations below with substitution and state the type of system.
1. \[
\begin{align*}
y &= x + 4 \\
3x + y &= 12
\end{align*}
\]
4.2: Solve Systems of Linear Equations by Substitution

Solve the system of equations below with substitution and state the type of system.

2. \[
\begin{align*}
6x - 3y &= 6 \\
2x - y &= 2
\end{align*}
\]

3. \[
\begin{align*}
x - 4y &= 8 \\
\frac{x}{3} - y &= 2
\end{align*}
\]

Application:
4. Dominique has four times as much money in his savings account as in his checking account. The total amount is $2,300.

Variables:

Equation 1:

Equation 2:
4.2: Solve Systems of Linear Equations by Substitution

Concept Check:
Solve the system of equations below with substitution and state the type of system.

5. \[
\begin{align*}
-x + 3y &= 6 \\
y &= \frac{1}{3}x + 4
\end{align*}
\]

6. \[
\begin{align*}
3x - y &= 6 \\
-4x + 2y &= -8
\end{align*}
\]
Solving Systems of Equations by Addition — p306

Step 1: Rewrite each equation in standard form, $Ax + By = C$.
Step 2: If necessary, multiply one or both equations by a nonzero number so the coefficients of a chosen variable in the system are opposites.
Step 3: Add the equations.
Step 4: Find the value of one variable by solving the resulting equation from Step 3.
Step 5: Find the value of the second variable by substituting the value found in Step 4 into either of the original equations.
Step 6: Check the proposed solution in the original system.

Example Problem Worked Out:

\[
\begin{align*}
2x - y &= 6 \\
-x + 4y &= 18
\end{align*}
\]

\[
\begin{align*}
2x - y &= 6 \\
2(-x + 4y &= 18) \quad \text{Multiply the second equation by 2.}
\end{align*}
\]

\[
\begin{align*}
2x - y &= 6 \\
-2x + 8y &= 36 \\
0x + 7y &= 42 \quad \text{The coefficients of } x \text{ are opposites, so now add the equations.}
\end{align*}
\]

\[
\begin{align*}
7y &= 42 \\
y &= 6 \\
2x - 6 &= 6 \\
2x &= 12 \\
x &= 6
\end{align*}
\]

Solve the equation for $y$.

Substitute $y = 6$ into the first equation and solve for $x$.

The solution to the system is $(6, 6)$.

Solve the systems of equations below with the Addition Method.

1. \[
\begin{align*}
3x - y &= 6 \\
-4x + 2y &= -8
\end{align*}
\]

2. \[
\begin{align*}
2x - 3y &= 5 \\
-4x - 7 &= -6y
\end{align*}
\]
3. \[
\begin{align*}
\frac{3x}{4} - \frac{y}{2} &= 1 \\
x - 5y &= 4
\end{align*}
\]

4. \[
\begin{align*}
0.4x - 0.8y &= 3.6 \\
0.3x - 0.6y &= 2.7
\end{align*}
\]

Student Practice:

5. \[
\begin{align*}
2x - 3y &= 5 \\
5x - 6 &= -2y
\end{align*}
\]
Application:
6. In recent years, the number of newspapers printed as morning editions has been increasing and the number of newspapers printed as evening editions has been decreasing. The number, \( y \), of daily morning newspapers in existence from 1997 through 2007 is approximated by the equation \( 146x - 10y = -7086 \), where \( x \) is the number of years since 1997. The number of daily evening newspapers is approximated by \( 111x + 5y = 4058 \).

a. Use the addition method to solve this system of equations.

\[
\begin{align*}
146x - 10y &= -7086 \\
111x + 5y &= 4058
\end{align*}
\]

b. Interpret your solution from part (a).

c. How many of each type of newspaper were in existence that year?
4.4: Applications of Systems of Equations

1. The tickets to a hockey game cost $69 for three adults and four children. For just two adults and one child the cost is $31. What is the cost of each ticket?

   Label the Variables.
   \[ x = \]
   \[ y = \]

   Write a system of equations.

   Solve the problem with your system.

   State your answer.

2. Joan Gundersen rented the same car model twice from Hertz, which rents this car model for a daily fee plus an additional charge per mile driven. Joan recalls that the car rented for 5 days and driven for 300 miles cost her $178, while the same model car rented for 4 days and driven for 500 miles cost $197. Find the daily fee and the mileage charge.

   Label the Variables.
   \[ x = \]
   \[ y = \]

   Write a system of equations.

   Solve the problem with your system.

   State your answer.
3. How many liters each of 40% acid solution and a 56% acid solution must be used to produce 60 liters of a 45% acid solution?

Label the Variables.

\[ x = \]

\[ y = \]

Write a system of equations.

Solve the problem with your system.

State your answer.

4. Macadamia nuts cost an astounding $16.50 per pound, but research by an independent firm says that mixed nuts sell better if macadamias are included. The standard mix costs $9.25 per pound. Find how many pounds of macadamias and how many pounds of the standard mix should be combined to produce 40 pounds that will cost $10 per pound. Find the amounts to the nearest tenth of a pound.

Label the Variables.

\[ x = \]

\[ y = \]

Write a system of equations.

Solve the problem with your system.

State your answer.
A linear inequality of two variables is visually solved by graphing. The graph below represents all the solutions for $y > 2x - 1$. All the ordered pairs in the shaded region are solutions to $y > 2x - 1$.

A solution to a system of inequalities is an ordered pair that satisfies all inequalities in the system. A graphed system is a visual way to show all the solutions.

<table>
<thead>
<tr>
<th>Boundary Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\leq, \geq$ solid (includes all points on the line)</td>
</tr>
<tr>
<td>$&lt;, &gt;$ dashed (excludes all points on the line)</td>
</tr>
</tbody>
</table>

**Example Problems:**

1. \[
\begin{align*}
\{ & y \leq 3x \\
& x - 2y \leq 8 
\end{align*}
\]

2. \[
\begin{align*}
\{ & x + 1 > 6 \\
& 2x \leq -10 
\end{align*}
\]
9.3: Solve Systems of Linear Inequalities

3. \[
\begin{aligned}
4x - y &\geq -2 \\
2x + 3y &\leq -8 \\
y &< -5
\end{aligned}
\]

Concept Check

4. \[
\begin{aligned}
-x + y &> 4 \\
y &< \frac{1}{2}x + 6
\end{aligned}
\]

5. \[
\begin{aligned}
x &\geq 3 \\
4x + 3y &< 12 \\
y &> -2 \\
y &< 0
\end{aligned}
\]
5.1 Exponents, Properties

Exponents provide an alternative way to write multiplication of the same number.

**Vocabulary**
- **Base** is the number that is multiplied.
- **Exponent** (or power) is how many times the base appears in the multiplication problem.

\[ x \cdot x \cdot x = x^3 \quad (5)(5) = \]

*The silent exponent on numbers and variables is 1! 2 is the same as \(2^1\)*

**Evaluating Expressions:**
Substitute the given value in and follow the order of operations.

\[ 2x^4 \text{ when } x = -1 \quad \frac{5}{x^2} \text{ when } x = 6 \quad -2xy^3 \text{ when } x = 2 \text{ and } y = -2 \]

What is the meaning and answer for each expression?

\[ (-2)^2 \text{ vs. } -2^2 \]

**Introducing the Product Rule**
Write out the meaning of each problem and then the simplified expression.

\[ x^3 \cdot x^4 \quad 3^2 \cdot 3^3 \]

**Product Rule** \(a^m \cdot a^n = a^{m+n}\)
- Add the exponents
- Keep the bases the same

Examples: Simplify the following expressions.

\[ x^2 \cdot x^3 \cdot x^4 \quad a^3 \cdot a \quad (ab^2)(a^2b^3) \]

\[ (5x^2) \cdot (2x) \quad (-2)^2 \cdot (-2)^3 \quad (2x)(3x^2) \]
5.1 Exponents, Properties

Introducing the Power Rule:
Write out the meaning of each problem and then the simplified expression.
\[(x^3)^2 \quad (x^2)^4\]

Power Rule \((a^n)^m = a^{nm}\)
- Multiply the exponents.
- Keep the base the same.

\[(a^5)^{10} \quad (z^6)^3\]

Product to a Power: Distribute the exponent outside the parenthesis with the Power Rule.
\[(x^5y)^3 = x^{15}y^3 \quad (2x^2y^3)^4 \quad (x^2y^3)^2\]

Quotient to a Power: Distribute the exponent outside the parenthesis with the Power Rule.
\[\left(\frac{x^3}{4y^2}\right)^4 \quad \left(\frac{-2x^3}{4y^5}\right)^3\]

Introducing the Quotient Rule:
Write out the meaning of each problem and then the simplified expression.
\[\frac{x^3}{x^2} = \frac{2^5}{2} =\]

Quotient Rule \(\frac{a^m}{a^n} = a^{m-n}\)
- Subtract the exponents (top – bottom exponent).
- Keep the bases the same.

\[\frac{x^{100}}{x^{95}} \quad \frac{10x^{100}}{2x^{50}} \quad \frac{4^3 x^5y}{4 x^3y}\]

Zero exponent rule:
Rule: Anything to the zero power equals 1 or \(a^0 = 1\).
\[2^0 \quad x^0 \quad (7a)^0 \quad 7a^0 \quad 6^2 - 6^0\]
5.2 Negative Exponents and Scientific Notation

Negative Exponent Rule: $a^{-m} = \frac{1}{a^m}$ or $\frac{1}{a^{-m}} = a^m$

Negative exponents indicate that a reciprocal needs to be taken. Negative exponents move the base to the other part of the fraction. Once the reciprocal has been taken (based moved to the other part of the fraction) remember to remove the negative.

Problems worked out.

\[
\frac{x^{-4}}{y^{-3}} = \frac{y^3}{x^4} \quad 2^{-2} = \frac{1}{2^2} = \frac{1}{4}
\]

Examples

\[
x^4 \quad 5^{-2} \quad 3x^{-3}
\]

Problems involving some or all of the exponent rules we have learned.

To simplify expressions: (These can be done in any order)

1. Remove parentheses using the power rule
2. Move the base of a negative exponent to the other part of the fraction
3. Simplify by reducing the fraction with product and quotient rules

\[
\left( -\frac{1}{3} \right)^{-4} \quad \frac{-1}{a^{-2}} \quad \frac{a^{-2}}{a}
\]

\[
2^{-2} + 2^{-1} \quad \frac{x^{-2}}{y^{-7}} \quad \frac{x^4x^3}{x^{10}}
\]

\[
\frac{(a^2)^3a}{(a^3)^4} \quad (x^2y^3)^4 \quad \left( \frac{2a}{b} \right)^{-3}
\]

\[
\left( \frac{x^2y^{-2}}{4y^{-5}} \right)^5 \quad \frac{2^3 x^{-3}y}{2^4x^2 y^{-3}}
\]
5.2 Negative Exponents and Scientific Notation

**Scientific notation** allows you to write large and very small numbers in a shorthand way. It is always written as a multiplication problem such as: $2.5 \times 10^3$ or $3 \times 10^{-4}$

1. The first number is always between 1 and 10
2. The second number is always a power of 10 (positive and negative powers)

**Write standard notation in scientific notation:**
1. Write the number between 1 and 10 as the first number
2. Count the number of places you have to move the decimal point to get the original number
3. Write this as the power of 10 (small number: negative exponent, large number: positive exponent)

<table>
<thead>
<tr>
<th>Number</th>
<th>Scientific Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>$3 \times 10^3$</td>
</tr>
<tr>
<td>50000000</td>
<td>$5 \times 10^7$</td>
</tr>
<tr>
<td>0.0005</td>
<td>$5 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.000254</td>
<td>$2.5 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

**Write scientific notation numbers in standard notation:**
Move the decimal the number of times the exponent indicates
- Negative exponent: create a small number by moving the decimal left
- Positive exponent: create a large number by moving the decimal right

<table>
<thead>
<tr>
<th>Scientific Notation</th>
<th>Standard Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.6 \times 10^2$</td>
<td>260</td>
</tr>
<tr>
<td>$5.1 \times 10^{-2}$</td>
<td>0.051</td>
</tr>
<tr>
<td>$3.1 \times 10^1$</td>
<td>31</td>
</tr>
<tr>
<td>$3.45 \times 10^{-5}$</td>
<td>0.0000345</td>
</tr>
</tbody>
</table>

Application: How many light years is 45 million kilometers if one light year is 10 trillion kilometers?
5.3 Polynomials, Vocabulary, Evaluating, Like Terms and Simplifying

The general name for this: \( x^2 - 3x + 6 \) is an algebraic expression.

A term is a part of an expression separated by an addition or subtraction sign.

Coefficient is the number multiplied with the variable and is located at the beginning of each term.

Degree of a term is the variable’s exponent.

Degree of a polynomial is the highest degree of all the terms. (Don’t add the powers together!)

Complete the table to express the terms, coefficients, and degree of each term in the expression \( x^2 - 3x + 6 \).

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*6 is more appropriately referred to as the ‘constant’ as it is not multiplied with a variable.

Classifications of Polynomials
- **Monomials** – polynomial with one term such as \( xy \) or \( 3x \) or \( -3 \)
- **Binomials** – polynomial with two terms such as \( x + y \) or \( 2x^2 + 5x \)
- **Trinomials** – polynomial with three terms such as \( 2x + 3y - 4z \)
- Polynomials with more than three terms are called ‘polynomials’ or ‘none of the above.’

Identify the following as monomial, binomial, or trinomial. Then state the degree of each polynomial.

\( 2x^3 \)

\( -3x^2 + 4x \)

\( \frac{1}{2} x^2 - 3x + 6 \)

Evaluating Polynomials:
To evaluate a polynomial, substitute the value provided for \( x \) and follow the order of operations.

\( x + 5; \quad x = 3 \)

\( x^2 + x + 1; \quad x = 5 \)

\( x^4 + x^2 - x^2 + 1; \quad x = -2 \)
5.3 Polynomials, Vocabulary, Evaluating, Like Terms and Simplifying

Like terms: terms having matching variable and the same exponents

Example: $4x^2$ and $\frac{1}{2}x^2$  
Non-Example: $2x$ and $3y$  
Non-Example: $x^3$ and $7x$

Simplifying polynomials: To simplify polynomials, add and subtract like terms.
- Add and subtract the coefficients
- Keep the variable and the exponent the same

\[
y + 3y \quad 4x^2 + x - 2x^2
\]

\[
-3x^2 + 5x + 5 - x^2 - 6x - 11 \quad 2ab + 3b - 4a + 10a - 5ab
\]

\[
7x^2 + 3xy^2 - 5x^2y + 6xy^2 - 7x^2y + y^2
\]

What is the difference between evaluating and simplifying polynomials?

Descending Powers with No Missing Terms

Problem Worked Out:
If I am given $2x^3 + 6$, the missing terms are an $x^2$ and an $x$ term. When the missing terms are inserted in the expression, then $2x^3 + 6$ becomes $2x^3 + 0x^2 + 0x + 6$.

Write the expressions below with all of the missing terms written.

$3x^3 - 4x$  
$x^5 - 3$
Add Polynomials: combine all like terms
  - Combine like terms using addition and subtraction.
    a. Add or subtract the coefficients
    b. Keep the variables and exponents the same.

Problem Worked Out
\[
(4x^3 - 6x^2 + 2x + 7) + (5x^3 - 2x)
\]
\[
4x^3 - 6x^2 + 2x + 7 + 5x^3 - 2x
\]
\[
4x^3 - x^2 + 7
\]

Example
\[
(3x^5 - 7x^3 + 2x - 1) + (3x^3 - 2x)
\]

Concept Check
\[
(x^2 + 2x - 3) + (4x^2 - 3x - 1)
\]

Subtracting Polynomials:
  - Distribute the negative sign to the polynomial to the right of the subtraction sign.
  - Combine like terms using addition and subtraction.
    a. Add or subtract the coefficients
    b. Keep the variables and exponents the same.

Problem Worked Out
\[
(5x - 3) - (2x - 11)
\]
\[
5x - 3 - 2x + 11
\]
\[
3x + 8
\]

Example
\[
(4x^3 - 10x^2 + 1) - (-3x^3 + x^2 - 11)
\]

Concept Check
\[
(4x^2 - 3x + 1) - (5x^2 - 3x + 7)
\]

\[
(7a^2 - 3b^2 + 10) - (-2a^2 + b^2 - 12)
\]
5.4 Add and Subtract Polynomials

Adding and Subtracting Polynomials with Word Expressions
Subtract \((8x^2 - 6)\) from \((7x^2 + 2x + 3)\).

**Concept Check**
Find the sum of \((2x - 9)\) and \((5x^2 - 7x + 16)\)

Adding and Subtracting Polynomials of Several Variables
- Combine like terms using addition and subtraction.
  a. Add or subtract the coefficients
  b. Keep the variables and exponents the same.

\[(x + y) + (x^2 + x - 4y)\]

\[(a^2 - ab + 4b^2) - (6a^2 + 8ab - b^2)\]

**Concept Check**
\[(4x^2 + y^2 + 3) - (5x^2 + y^2 - 2)\]
5.5 Multiplying Polynomials

Multiplying monomials

\[-2x^3(3x^2)\]
\[(-x^3)(-x^6)\]

Distributive property \(a(b + c) = ab + ac\)

\[3x(4x^2 - 5)\]
\[\frac{1}{2}(x^2 - 2x + 5)\]

\[2x^2(-3x^2 - 4x + 5)\]
\[4xy^2(7x^3 + 3x^2y^2 - 9y^3)\]

Multiplying Binomials:
Multiply each term of the first polynomial by each term of the second polynomial, then combine like terms.

Example Worked Out in Two Different Visual Methods

\[(3x + 1)(2x - 5)\]
\[3x(2x - 5) + 1(2x - 5)\]
\[6x^2 - 15x + 2x - 5\]
\[6x^2 - 13x - 5\]

\[(3x + 1)(2x - 5)\]
\[3x(2x) + 3x(-5) + 1(2x) + 1(-5)\]
\[6x^2 - 15x + 2x - 5\]
\[6x^2 - 13x - 5\]

\[(1 - 2x)(1 - 3x)\]
\[(3x^2 + 2)(2x^2 - 1)\]

\[(2x + y)(3x - y)\]

\[\text{Concept Check}\]
\[(2x + 1)(-3x - 5)\]

\[(x + 2y)^2\]

\[\text{Concept Check}\]
\[(4a)^2 + (3b)^2 \quad \text{versus} \quad (4a + 3b)^2\]
5.5 Multiplying Polynomials

Multiplying Polynomials
\((x^2 - 3x + y)(x^2 + 2x - y)\)

Concept Check
\((x + 1)(3x^2 - 2x + 1)\)

Optional: multiply vertically
Be sure to align like terms (see section 5.5 example 10 on p377)

\[-2x^2 + 7x + 1\]
\[3x - 6\]

HW: Do you prefer multiplying vertically or horizontally? Why? What would be the advantage to the vertical method?
Binomial Squared: A binomial squared can be multiplied out by using the distributive property or FOIL Method used in prior lessons. Today you will use a ‘short-cut’ technique.

Students Multiply: \((3x + 4)^2\)

Teacher Explain:

\[
(3x + 4)^2
= \underbrace{3x \times 3x}_{\text{square}} + \underbrace{4 \times 4}_{\text{square}} + \underbrace{2 \times 3x \times 4}_{\text{double}}
= 9x^2 + 12x + 16
\]

Binomial Squared = Perfect Square Trinomial
Square the first term, twice the product of both terms, square the second term
\((a + b)^2 = a^2 + 2ab + b^2\)

\[(x - 3)^2\]

\[(5x - 1)(5x - 1)\]

\[(y - \frac{2}{3})^2\]

\[(3x - 6y)^2\]

Helpful Hint
Notice that
\((a + b)^2 = a^2 + 2ab + b^2\) *The middle term, \(2ab\), is missing.
\[(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2\]

Likewise,
\((a - b)^2 = a^2 - 2ab - b^2\) *The middle term, \(-2ab\), is missing.
\[(a - b)^2 = (a - b)(a - b) = a^2 - 2ab - b^2\]

Product of a Sum and Difference
Students Multiply: \((2x + 5)(2x - 5)\)

Teacher: \((2x + 5)(2x - 5)\)

\[4x^2 - 25\]
5.6 Special Products – Binomial Squared & Product of a Sum and Difference

**Product of a Sum and Difference = Difference of Two Squares:**
First term squared minus second term squared

\[(a + b)(a - b) = a^2 - b^2\]

\[(3x + 2)(3x - 2) \quad (2x - 3y)(2x + 3y)\]

**Variety of Special Products**

\[(7x - 1)^2 \quad \left(5y - \frac{1}{9}\right)\left(5y + \frac{1}{9}\right)\]

**Concept Check**

\[(5y - 3)(5y + 3) \quad (2x + 1)^2\]

\[(4y + 3)(2y - 3)\]

**Answer each exercise as true or false.**

\[(x + 3)^2 = x^2 + 16\]

\[(x + 4)(x - 4) = x^2 + 16\]

For \((2x - 1)(x + 6)\), the product of the first terms is \(2x^2\).

The product of \((x - 1)(x^3 + 3x - 1)\) is a polynomial of degree 5.

**Match each expression on the left to the equivalent expression on the right.**

1. \((a - b)^2\)  
   a. \(a^2 - b^2\)

2. \((a - b)(a + b)\)  
   b. \(a^2 + b^2\)

3. \((a + b)^2\)  
   c. \(a^2 - 2ab + b^2\)

4. \((a + b)^2(a - b)^2\)  
   d. \(a^2 + 2ab + b^2\)

   e. none of these
5.6 Special Products – Binomial Squared & Product of a Sum and Difference

Your yard is a square with dimensions \((x + 3)\) by \((x + 3)\). What is the area of the yard?

\[
\begin{array}{|c|}
\hline
(x + 3) \text{ ft} \\
\hline
(x + 3) \text{ ft} \\
\hline
\end{array}
\]

Your pool is a rectangle with dimensions \((x + 5)\) by \((x - 5)\). What is the area of the pool?

\[
\begin{array}{|c|}
\hline
(x - 5) \text{ ft} \\
\hline
(x + 5) \text{ ft} \\
\hline
\end{array}
\]
6.1 Greatest Common Factor and Factor by Grouping

**Greatest Common Factor (GCF)** is the largest number that divides into a set of numbers.

Find the GCF of each set:
- 6,12,36
- 15,30,27

With variables, the GCF is the variable raised to the smallest exponent in the list.

Find the GCF of each set:
- $3x, 12x^2, 9x^3$
- $6x^5, 12x^4, 9$

**Concept Check**
- $14x^2, 8x^3, 12x^5$

**Factoring** is the process of rewriting a polynomial as a multiplication problem.

1. Find the GCF that goes into the polynomial. If the polynomial begins with a negative number, use the negative in the GCF. Place the GCF on the left of the parentheses. $\rightarrow$ GCF (remainder)
2. Write the remainder of your division in parentheses and place it next to the GCF

True or False?
- $3x + 15 = 3(x + 5)$
- $2x^2 - 4x = 2x(2x - 2)$

Factor out the GCF from each polynomial:
- $21x - 7$
- $3x^2 + 6x + 3$

**Concept Check**
- $5x - 25$
- $5x^2 - 15x$

**Factoring a Negative Out of an Expression**
Factor -1 from each
- $-x - 5$
- $-k + 7$

**Concept Check**
- $-x - y + 3$
6.1 Greatest Common Factor and Factor by Grouping

Greatest Common Factor with Several Variables
*The variable raised to the smallest exponent in the list
Example: $4ab^3 - 7ab^2 + 5b = b(4ab^2 - 7ab + 5)$

Factor out the GCF from each polynomial
$-6x^3y^2 + 3x^2y^3 - 18xy^4$

Concept Check
$24x^3y^3 + 12xy^2 - 36x^2y$

Factoring by grouping
1. Look for a common factor among all the terms $\rightarrow$ GCF (remainder)
2. Pair the terms
3. Find the GCF in the first two terms
4. Find the GCF in the last two terms
5. Write the GCFs in one parenthesis and write the common parenthesis ONE TIME in another parenthesis
6. Check the new parentheses for more factoring

Here's what some of the problems look like:
$ab - 5a + cb - 5c$  \hspace{1cm} $x - 3xy + 2z - 6zy$

$x(a+b) - y(a+b) = (a+b)(x-y)$  \hspace{1cm} $2x(c+d) + 3(c+d)$

Concept Check
$x^3 + 6x^2 - 2x - 12$  \hspace{1cm} $3x(a - c) + (a - c)$

$2x^2(a^2 + a + 5) - 5(a^2 + a + 5)$
6.2 factoring trinomials \( ax^2 + bx + c \) where \( a = 1 \)

Factoring is figuring out what multiplication gave the polynomial expression you are working with.

\[
\begin{align*}
\text{Above } x^2 + 5x + 6 & \text{ became } (x + 2)(x + 3). \text{ How is the middle term } 5x \text{ important?} \\
\text{Above } x^2 - 3x + 2 & \text{ became } (x - 2)(x - 1). \text{ How is the middle term } -3x \text{ important?}
\end{align*}
\]

Example: Factor \( x^2 - 4x + 12 \).

<table>
<thead>
<tr>
<th>Factors of ( x^2 )</th>
<th>Factors of -12</th>
<th>Which pair of factors adds to -4?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x, x )</td>
<td>-1, 12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1, -12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-2, 6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2, -6</td>
<td>(-8) So the answer is ((x + 2)(x - 6)).</td>
</tr>
<tr>
<td></td>
<td>-3, 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3, -4</td>
<td></td>
</tr>
</tbody>
</table>

Steps for Factoring

1. Arrange the polynomial in descending order.
2. Find the GCF that goes into the polynomial. If the polynomial begins with a negative number, use the negative in the GCF. Place the GCF on the left of the parentheses. \( \rightarrow \) GCF (remainder)
3. List all factors of the first term
4. List all factors of the last term and decide which pairing creates the middle term.
5. Write the parentheses and use the correct signs.
   - Last number is positive \( \rightarrow \) both signs will be the same as the middle term
   - Last number is negative \( \rightarrow \) one sign is positive and one is negative
6. Check the answer.

Factor the following

\[
\begin{align*}
\text{x}^2 + 7x + 12 & \quad \text{x}^2 + 8x - 20 \\
x^2 - 8x + 16 & \quad x^2 - 5x + 4
\end{align*}
\]

Concept Check

\[
\begin{align*}
\text{x}^2 - 10x - 9 & \quad \text{y}^2 - 15y + 54
\end{align*}
\]
6.2 factoring trinomials $ax^2 + bx + c$ where $a = 1$

Additional Factoring Problems (watch for the GCF)

$-225x^2 - 20x^3 + 5x^4$  

$-x^2y + 26xy - 48y$

$-x^2 - 3x - 2$  

$7a^3b - 35a^2b^2 + 42ab^3$

Concept Check

$2x + x^2 + 1$

$x^3 - 2x^2 - 24x$

$a^2 - 13ab + 30b^2$
6.3 factoring $ax^2 + bx + c$ where $a \neq 1$

When factoring trinomials that have a number other than 1 in front of the $x^2$, such as $3x^2 + 7x + 2$, there are several methods you can use. I will model one for the class. *If after two days this method does not work for you, please see me for additional methods.*

**Steps for Factoring with the Trial and Error Method**
1. Arrange the polynomial in descending order.
2. Find the GCF that goes into the polynomial. If the polynomial begins with a negative number, use the negative in the GCF. Place the GCF on the left of the parentheses. $\Rightarrow$ GCF (remainder)
3. List all factors of the first term
4. List all factors of the last term
5. Set up two parentheses $GCF(\ ) (\ )$
6. There are usually several ways the numbers can be placed so you MUST FOIL, use the distributive property, to check your answer. When you multiply your “guess” in the parentheses, it has to equal the original problem.

Factoring is figuring out what multiplication gave the polynomial expression you are working with.

\[
\begin{align*}
4x^2 + 9x + 5 & = (4x + 5)(x + 1) \\
4x^2 + 12x + 5 & = (2x + 5)(2x + 1)
\end{align*}
\]

Above $4x^2 + 9x + 5$ became $(4x + 5)(x + 1)$. How is the middle term $9x$ important?

Above $4x^2 + 12x + 5$ became $(2x + 5)(2x + 1)$. How is the middle term $12x$ important?

**Example: Factor $7x^2 - 53x + 28$.**

<table>
<thead>
<tr>
<th>Factors of $x^2$</th>
<th>Factors of 28</th>
<th>Parentheses created</th>
<th>Polynomials the Parentheses Create</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7x, 1x$</td>
<td>28, 1</td>
<td>$(7x + 28)(x + 1)$</td>
<td>$7x^2 + 7x + 28x + 28$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(7x + 1)(x + 28)$</td>
<td>$7x^2 + 196x + x + 28$</td>
</tr>
<tr>
<td>$-28, -1$</td>
<td></td>
<td>$(7x - 28)(x - 1)$</td>
<td>$7x^2 - 7x - 28x + 28$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(7x - 1)(x - 28)$</td>
<td>$7x^2 - 196x - x + 28$</td>
</tr>
<tr>
<td>$14, 2$</td>
<td></td>
<td>$(7x + 14)(x + 2)$</td>
<td>$7x^2 + 14x + 14x + 28$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(7x + 2)(x + 14)$</td>
<td>$7x^2 + 98x + 2x + 28$</td>
</tr>
<tr>
<td>$-14, -2$</td>
<td></td>
<td>$(7x - 14)(x - 2)$</td>
<td>$7x^2 - 14x - 2x + 28$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(7x - 2)(x - 14)$</td>
<td>$7x^2 - 98x - 2x + 28$</td>
</tr>
<tr>
<td>$7, 4$</td>
<td></td>
<td>$(7x + 7)(x + 4)$</td>
<td>$7x^2 + 28x + 7x + 28$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(7x + 4)(x + 7)$</td>
<td>$7x^2 + 49x + 4x + 28$</td>
</tr>
<tr>
<td>$-7, -4$</td>
<td></td>
<td>$(7x - 7)(x - 4)$</td>
<td>$7x^2 - 28x - 7x + 28$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(7x - 4)(x - 7)$</td>
<td>$7x^2 - 49x - 4x + 28$</td>
</tr>
</tbody>
</table>

Of the above 12 possible factor combinations, which ONE creates the middle term of -53?

So $7x^2 - 53x + 28$ factors as __________________________
6.3 factoring $ax^2 + bx + c$ where $a \neq 1$

Factor the following polynomial expressions

$3x^2 + 7x + 2$  $8x^2 - 14xy + 3y^2$

$6x^3 - 22x^2 + 12x$  $3a^2b^2 + 16ab - 12$

**Concept Check**

$3x^2 + 5x + 2$  $49a^2 - 7a - 2$

$5x^2 - 12x + 4$  $8a^3 + 5a^2 - 3a$
Factoring Perfect Square Trinomials refers to trinomials where the first and last terms are perfect squares. A perfect square trinomial factors into a binomial squared.

\[ 9x^2 + 24x + 16 = (3x + 4)^2 \]

Further Examples:
\[ x^2 + 8x + 16 = (x + 4)^2 \]  
\[ 25x^2 - 10xy + y^2 \]

\[ 2x^2 - 8xy + 8y^2 = 4(x^2 - 2xy + 2y^2) \]

\[ 4x^2 + 20x + 9 \]

Concept Check
\[ 4x^2 + 12x + 9 = (2x + 3)^2 \]

\[ x^2y^2 - 6xy + 9 \]

\[ 4r^2 - 17tr + 4r^2 \]

Notice that
\[ a^2 + 2ab + b^2 = (a + b)^2 \]
\[ a^2 + b^2 \text{ The middle term, } 2ab \text{ is missing. This DOES NOT factor.} \]
Likewise,
\[ a^2 - 2ab + b^2 = (a - b)^2 \]
\[ a^2 - b^2 \text{ The middle term, } 2ab \text{ is missing. This DOES factor.} \]

We will now explore this pattern on the next page.
6.5 Factor Special Cases

Factoring a Difference of Squares *No middle term
\[ x^2 - 25 \quad 4x^2 - 49 \]

Factoring the difference of two squares Factoring the difference of two squares results in the product of a sum and difference. The rules are:
1. Factor the greatest common factor out, leaving remainder in a parenthesis
2. Must be a subtraction problem
3. The two terms must be perfect squares
4. Factor as follows: \( a^2 - b^2 = (a+b)(a-b) \)

Example:
\[ x^2 - 16 = \quad 2x^4 - 18y^2 \]

\[ -a^4 + b^2 \quad x^2y^2 - 4a^2b^2 \]

Concept check
\[ 36x^2 - 9y^2 \quad 4x^3 - 49x \]

Is \((x - 4)(x^2 - 9)\) completely factored?

Table from previous page

\[ a^2 + 2ab + b^2 = (a + b)^2 \]
\[ a^2 + b^2 \quad *\text{The middle term, } 2ab. \text{ is missing. This DOES NOT factor.} \]
\[ a^2 - 2ab + b^2 = (a - b)^2 \]
\[ a^2 - b^2 = (a - b)(a + b) \]
We have learned to factor several different forms of polynomials. Here is a summary of what you have learned:

Special Cases
- **Perfect Square Trinomial** = Binomial Squared
  \[ a^2 + 2ab + b^2 = (a + b)^2 \]
  \[ a^2 - 2ab + b^2 = (a - b)^2 \]
- **Difference of Squares**
  \[ a^2 - b^2 = (a - b)(a + b) \]

Non-special cases
- \( ax^2 + bx + c \) where \( a = 1 \)
  1. List all factors of the last term and decide which pairing creates the middle term.
  2. Write the parentheses and use the correct signs. \( GCF(-)(-) \)
     - Last number is positive \( \rightarrow \) both signs will be the same as the middle term
     - Last number is negative \( \rightarrow \) one sign is positive and one is negative
- \( ax^2 + bx + c \) where \( a \neq 1 \) (these are the time consuming problems)
  1. List all factors of the first term
  2. List all factors of the last term
  3. Set up two parentheses. \( GCF(-)(-) \)
  4. There are usually several ways the numbers can be placed so you MUST FOIL, use the distributive property, to check your answer. When you multiply your "guess" in the parentheses, it has to equal the original problem.

Prime
- \( a^2 + b^2 \) *The middle term, \( 2ab \) is missing. This DOES NOT factor.
  ANY polynomial that does not factor

Instructor: Model the following problems then have students complete many on their own.

\[
49x^2 - 42xy + 9y^2
\]
\[
x^4 - 10x^2 - 9
\]
\[
x^3 - 2x^2 + 3x - 6
\]
\[
4n^2 - 6n
\]
\[
x^2 + x - 12
\]
\[
20 - 3x - 2x^2
\]
6.1-6.5 Review of Factoring Methods

Concept Check

$x^2 - x - 30$  

$8a^2 + 6ab - 5b^2$

$12a^3 - 24a^2 + 4a$  

$2x^3 - 18x$

$7(x - y) + y(x - y)$  

$y^2 + 22y + 96$

$4x^2 - 2xy - 7yz + 14xz$  

$12x^2 + 34x + 24$

$28 - 13x - 6x^2$  

$6y^2 + y - 15$
6.6 solving quadratic equations by factoring

Zero Factor Property
1. If the result of multiplying two numbers together is zero, what do you know about the numbers?
2. If five times a number is zero, what is the unknown number?
3. If $-4x = 0$, what is the value of $x$?
4. If $(x)(17) = 0$, what is the value of $x$?
5. If $3(x - 10) = 0$, what is the value of $x - 10$? What is the value of $x$?
6. If $2x(15) = 0$, what is the value of $x$?
7. If $x(x - 1) = 0$, what is the value of $x$?
8. If $(x + 2)(3x + 1) = 0$, what is the value of $x$?

Zero Factor Property: If $ab = 0$, then $a = 0$ or $b = 0$

Solving Linear Equations
Put the thoughts about the Zero Factor Property on hold for a minute. We need to review how to solve linear equations, which we did back in Chapter 2.

Solve for $x$:
1. Move the constant to the other side using the addition property of equality.
2. Isolate the variable using the multiplication property of equality

Solve for $x$

\[
\begin{align*}
2x &= 0 & x - 5 &= 0 & -x + 6 &= 0 & 2x - 5 &= 0
\end{align*}
\]

Solving Quadratic Equations:
Now we will put the two ideas, zero factor property and solving linear equations, together to solve quadratic equations.

1. Arrange the polynomial so that it equals 0 and the terms are in descending order. (this is standard form)
2. Factor out the GCF and leave the remainder in parenthesis $GCF(\text{remainder})$
3. Factor the quadratic expression $GCF(\quad)$
4. Set each factor equal to 0 and solve for $x$ $GCF=0$ or $(\quad)=0$ or $(\quad)=0$

Example problem worked out:

\[
\begin{align*}
x^2 - 8x &= -7 \\
x^2 - 8x + 7 &= 0 & \text{Use the additive property of equality to set the equation equal to zero. Add 7 to both sides} \\
(x - 7)(x - 1) &= 0 & \text{Factor the quadratic expression} \\
x - 7 &= 0, \ x - 1 &= 0 & \text{Use the zero factor property to change the problem into two linear equations.} \\
\quad x &= 7 \quad \text{or} \quad x = 1 & \text{Use the additive property of equality to solve for $x$.}
\end{align*}
\]
6.6 solving quadratic equations by factoring

Solve the following quadratic equations.

\[ 3x^2 - 9x = 0 \quad 10x^2 + 20x = 0 \]

\[ 3x^2 - 27 = 0 \]

\[ 2x^2 - 3x + 1 = 0 \]

Concept Check

Additional equations to solve. Remember to set the equation equal to zero and then factor!

\[ x^2 = 25 \quad x^3 - 8x^2 = -7x \]

\[ (x + 3)(x + 8) = x \]

What is a quadratic equation?

It is an equation where the highest power of \( x \) is squared. The standard form of quadratic equations is \( ax^2 + bx + c = 0 \).

There are several ways to solve quadratic equations. In M095 and M121 you will explore other ways to solve quadratic equations. But, for now factoring is THE WAY to solve quadratic equations. Just remember to set the equation equal to zero BEFORE factoring!

Concept Check

\[ x^2 - 10x = -16 \quad x(4x - 11) = 3 \]
Recommended Process

1. Draw and label a picture of the problem, or specifically label what $x$ is representing.
2. Use the appropriate formula to write an equation.
3. Rearrange the quadratic equation to equal zero.
4. Factor and solve for $x$ with the zero factor property.

The formulas for this section are:

- Area of a square with sides of length $x$ is $A = x^2$
- Area of a rectangle with sides of length $l$ and $w$ is $A = lw$
- Area of a triangle with base $b$ and height $h$ is $A = \frac{1}{2}bh$
- Area of a parallelogram with base $b$ and height $h$ is $A = bh$
- Area of a circle with radius $r$ is $A = \pi r^2$

The lengths of the sides of a right triangle $a$, $b$ and $c$ are related by $a^2 + b^2 = c^2$

1. A triangle has sides of length $2x$, $(2x + 5)$ and $(x^2 + 3)$. Its perimeter is 85 feet. Find each side length.
   
   a. Draw and label a picture or specifically label what $x$ is representing.

   b. Write an equation to solve the problem.

   c. Solve the equation.

   d. State the answer in a sentence.
2. The length of a rectangle is 9 inches more than its width and the area is 112 square inches. Find the dimensions of the rectangle.
   a. Draw and label a picture or specifically label what x is representing.
   b. Write an equation to solve the problem.
   c. Solve the equation.
   d. State the answer in a sentence.

Concept Check
4. The area of a rectangle is 84 square inches. Its length equals \((x + 3)\) and the width equals \((x - 2)\). Find the length and width.
   a. Draw and label a picture or specifically label what x is representing.
   b. Write an equation to solve the problem.
   c. Solve the equation.
   d. State the answer in a sentence.
3. Find the length of the shorter leg of a right triangle if the longer leg is 10 inches more than the shorter leg and the hypotenuse is 10 inches less than twice the shorter leg.

   a. Draw and label a picture or specifically label what x is representing.

   b. Write an equation to solve the problem.

   c. Solve the equation.

   d. State the answer in a sentence.

Concept Check
5. The sum of a number and its square is 132. Find the number.

   a. Draw and label a picture or specifically label what x is representing.

   b. Write an equation to solve the problem.

   c. Solve the equation.

   d. State the answer in a sentence.